

Nonlinear Control of Underactuated Systems using a 3-D Virtual Laboratory

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Abstract—Control of underactuated mechanical systems is currently one of the most active fields in research due to the diverse applications of these systems in real-life. The aim of this article is focused on the application of nonlinear control techniques for underactuated systems and the virtual simulation of their dynamics behavior. The main contribution of this research is related with the applications of balancing controllers designed with linearization techniques, and including swing-up control using energy based methods for two of the most typical underactuated systems used for testing nonlinear control: The cart-pole and the rotating pendulum systems. The second contribution relies in the development of a virtual laboratory for testing this algorithms and also with a great feature included; the platform is not tied to specific embedded controllers, the users can proof their own control techniques, adding control equations using a graphical user interface developed for that purpose. Finally, the analytical results will be validated via numerical solutions implemented on Matlab-Simulink toolbox, comparing the controllers and the simulation capabilities through several test cases.

Index Terms— Underactuated Systems, Feedback linearization, Swing-up control, Virtual Environments and Computer Graphics.

I. INTRODUCTION.

UNDERACTUATED systems are mechanical systems with more degrees of freedom than number of actuators. Control of underactuated systems is currently an active field in research due to their broad applications in robotics, aerospace and military vehicles [1]. The examples of underactuated systems include mobile robots, humanoids, robots on mobile platforms, snake-like and swimming robots, aircraft, spacecraft, helicopters, satellites, and underwater vehicles.

The scope of this article consists in the application of nonlinear control techniques in order to exemplify in detail how to apply them to control the cart-pole and the rotating pendulum systems [2]. The main control methods applied to examples of both inverted pendulums are based on swing-up of the pendulum from its downward position and then switching to a balancing controller that is designed using a linearization technique or gain scheduling to balance the pendulum [3]. The drawback of some existing surveys available [4] inside the dynamics modeling of these kind of systems relies in that in some cases, their dynamic model is not complete enough in order to consider the major physics

variables such as friction, inertial properties, motor models, etc. In other works, the dynamics equations are very complete, but they lack strong foundation related with the control techniques. Our approach, more than focused in the creation of novel control algorithms, consists in the analysis and the application of different control techniques from novel implementations, tuning or mixing them, in order to apply them using a complete dynamic description of the system via Euler-Lagrange or Newton formulations [5]. Then, our goal consists in applying existing swing-up design methods with a fast response consuming the less power as possible. In addition, we present asymptotic stabilization of both systems to their upright equilibrium points using nonlinear state feedback.

Furthermore, the other scope is related with the development of a 3-D virtual laboratory to model and to test the controllers. The process of optimizing and performing mathematical models has been increasingly aided by the advent of high performance computational virtual environments, with the purpose of implementing, modeling and simulating the physical behavior of these systems via numerical solutions [6]. This reason has inspired the creation of numerous graphical software environments, such as Pro/Engineer, Matlab-Simulink, Simnon, etc. However most of these packages are expensive, usually limited in the amount of available features and in some cases without a graphical environment of simulation with all the actual 3-D engines capabilities, for this reason, they lack realism to simulate the behavior of the systems.

Finally, our platform will bring to the user a great look and feel and it will turn into a virtual laboratory to testing controllers. For the validation of the results obtained, Matlab-Simulink toolbox will be used in order to compare numerical solutions and performance.

II. MODELING AND SIMULATION.

A. General Overview.

In this section, the nonlinear dynamics models and the control techniques to use will be presented, describing all the process in order to obtain the mathematical framework to model it, using the 3D virtual platform.

This platform named VISUNS: Visual Simulation of Underactuated Systems [7], is a multiplatform 3D user interface, compatible with the Linux, MacOSX and MS Windows operating systems, implemented with open-source tools. The main functional blocks of VISUNS have been

implemented using the Python programming language for data flow management among modules. These modules are composed by the *Graphical Engine* for solid rendering and modeling, the *Main Class* for data structuring and the *Simulation Engine*, including the dynamics modeling, the controller setup and the ODE solvers. All the critical code such as dynamics models, controllers and numerical integrators have been written with the C++ language, in order to obtain better performance for the real time simulation, due to python is an interpreted language. Section three will show the software design in detail.

B. Non-linear dynamic models.

The Cart-pole system consists of an inverted pendulum on a cart, while the rotating pendulum is an inverted pendulum on a rotating arm; both systems are depicted in Fig. 1 [8]

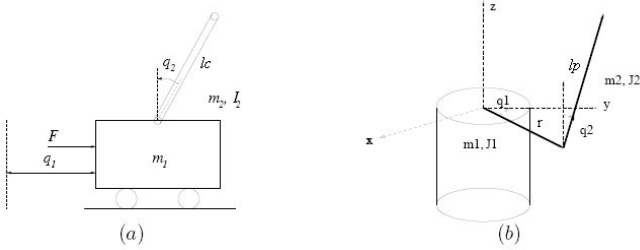


Fig. 1. (a) Cart-pole system and (b) rotating pendulum.

Referring to Fig. 1a and applying Newton's 2nd law at the center of mass of the pendulum along the horizontal and vertical components as shown in equation (1), yields:

$$\begin{aligned} V - m_2 g &= m_2 \frac{d^2}{dt^2} (l_c \cos(\theta)) \\ H &= m_2 \frac{d^2}{dt^2} (x + l_c \sin(\theta)) \end{aligned} \quad (1)$$

In equation (1), we have replaced q_1 from Fig. 1 by x , due to this joint is translational; and q_2 by θ . Taking moments in the center of gravity, yields the torque equation (2):

$$I\ddot{\theta} + c\dot{\theta} = Vl_c \sin(\theta) - Hl_c \cos(\theta) \quad (2)$$

Applying Newton's 2nd law for the cart in Fig. 1:

$$F - H = M\ddot{x} + b\dot{x} \quad (3)$$

Where m_1 is the mass of the cart and m_2 is the pendulum mass; l_c is the distance from the Center of Mass (CM) of the pendulum to the pivot; x is the horizontal displacement of the cart; g is the gravitational acceleration; q_2 is the joint angle replaced by θ ; b is the cart viscous friction coefficient; c is the pendulum viscous friction coefficient; I is the moment of inertia of the pendulum in the CM and F is the horizontal control force on the cart. Combining (1) to (3), the non-linear mathematical model of the cart and pendulum system is obtained and is given by (4).

$$\begin{aligned} \ddot{\theta} &= \frac{1}{I + l_c^2 m_2} [l_c m_2 (g \sin(\theta) - \ddot{x} \cos(\theta)) - c\dot{\theta}] \\ \ddot{x} &= \frac{1}{m_1 + m_2} [F - l_c m_2 (\ddot{\theta} \cos(\theta) - \dot{\theta}^2 \sin(\theta)) - b\dot{x}] \end{aligned} \quad (4)$$

The nonlinear model for the cart-pole system has been obtained. In addition, a dc-motor model has been included in order to obtain the relation between the applied torque and the voltage for the cart.

$$F = \frac{K_t K_g}{Rr} V - \frac{K_t^2 K_g^2}{Rr^2} \dot{x} \quad (5)$$

The parameter R is the motor armature resistance, r is the motor pinion radius, K_t is the motor torque constant, and K_g is the gearbox ratio. For the rotating pendulum nonlinear model, Euler-Lagrange formulations have been established, in order to obtain the lagrangian based on the difference between the kinetic (K) and potential (U) energy of the system. The lagrangian (L) is defined in equation (6) and solved by algebraic treatment in equation (7):

$$L(q, \dot{q}) = K(q, \dot{q}) - U(q) = \frac{1}{2} \dot{q}^T M(q) \dot{q} - U(q) = \frac{1}{2} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}^T \begin{bmatrix} ml_p^2 & mrl_p \cos(q_2) \\ mrl_p \cos(q_2) & I + mr^2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} - \begin{bmatrix} -mgl_p \sin(q_2) \\ 0 \end{bmatrix} \quad (6)$$

Solving the system: $M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = F(q)u$, given that $\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = F(q)u$ for $F(q) = \{f_{q1}, f_{q2}\}$:

$$\begin{aligned} \begin{bmatrix} \ddot{q}_2 \\ \ddot{q}_1 \end{bmatrix} &= \begin{bmatrix} ml_p^2 & mrl_p \cos(q_2) \\ mrl_p \cos(q_2) & I + mr^2 \end{bmatrix}^{-1} \left(-\frac{1}{2} \begin{bmatrix} 0 & -mrl_p \sin(q_2) \dot{q}_2 \\ -mrl_p \sin(q_2) \dot{q}_2 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} - \begin{bmatrix} -mgl_p \sin(q_2) \\ 0 \end{bmatrix} + \begin{bmatrix} f_{q2} \\ f_{q1} \end{bmatrix} \right) \\ \ddot{q}_1 &= \frac{\frac{1}{2} mrl_p \dot{q}_2^2 \sin(q_2) + \tau}{I + mr^2 (1 - \cos(q_2)^2)} - \frac{r \cos(q_2) \left(\frac{1}{2} q_2 \dot{q}_1 \sin(q_2) + mgl_p \sin(q_2) + b\dot{q}_2 \right)}{l_p (I + mr^2 (1 - \cos(q_2)^2))} \\ \ddot{q}_2 &= \frac{(I + mr^2) \left(\frac{1}{2} mrl_p \dot{q}_2 \dot{q}_1 \sin(q_2) + mgl_p \sin(q_2) + b\dot{q}_2 \right)}{ml_p^2 (I + mr^2 (1 - \cos(q_2)^2))} - \frac{r \cos(q_2) \left(\frac{1}{2} mrl_p \dot{q}_2^2 \sin(q_2) + \tau \right)}{l_p (I + mr^2 (1 - \cos(q_2)^2))} \end{aligned} \quad (7)$$

In equation (8) the dc-motor model for the rotating pendulum has been included, where m is the pendulum mass, l_p the length of the pendulum, I is the moment of inertia in the center of mass CM: r is the pivot length, b is the pendulum viscous friction coefficient, K_a is the torque constant, R_a the motor armature resistance and g the gravity constant.

$$\tau = \frac{K_a}{R_a} V - \frac{1}{R_a} \dot{q}_1 \quad (8)$$

C. Swing up and stabilizing controllers for the cart-pole and the rotating pendulum systems.

Equation (4) is used to model the dynamics behavior of the cart-pole with motor dynamics included in equation (5). The same nonlinear model was used to design swing-up controller, nevertheless, for the design of the linear state feedback controller used for the stabilization of the pendulum, it is necessary to obtain a linear model [9].

For the linearization of the nonlinear equation, we consider the following: when the pendulum is close to the equilibrium point, defined as $\theta=0$ and $\dot{\theta}=0$, then for small values of these variables, the $\sin(\theta) \approx \theta$, $\cos(\theta) \approx 1$ and $\dot{\theta}^2 \approx 0$.

Replacing these approximations in equation (4), the linear model in state form is: (note that the dc motor dynamics equation from (5) has been included in (9)).

$$\begin{aligned} X_1 &= x, X_2 = \dot{x}, X_3 = \theta, X_4 = \dot{\theta} \\ \dot{X} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -kv_2 - v_2 \frac{K_\tau^2 K_g^2}{Rr^2} & \frac{-(l_c m_2)^2 g v_2}{I + l_c^2 m_2} & \frac{l_c m_2 p v_2}{I + l_c^2 m_2} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{l_c m_2 k v_1}{m_1 + m_2} + l_c m_2 v_1 \frac{K_\tau^2 K_g^2}{Rr^2} & l_c m_2 g v_1 & -p v_1 \end{bmatrix} X \\ &+ \begin{bmatrix} 0 \\ v_2 \frac{K_\tau K_g}{Rr} \\ 0 \\ \frac{l_c m_2 v_1 K_\tau K_g}{(m_1 + m_2) Rr} \end{bmatrix} V \quad \text{and } Y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} X \quad \text{where,} \\ v_1 &= \frac{m_1 + m_2}{I(m_1 + m_2) + l_c^2 m_2 m_1} \quad y \quad v_2 = \frac{I + l_c^2 m_2}{I(m_1 + m_2) + l_c^2 m_2 m_1} \end{aligned} \quad (9)$$

As mentioned before, the inverted pendulum control was split in two main phases; the swing-up phase and the stabilizing phase [9]. The swinging up of a pendulum from the downward position can also be accomplished by controlling the amount of energy in the system. The energy in the pendulum system can be driven to a desired value through the use of feedback control. By adding enough energy such that its value corresponds to the upright position, the pendulum can be swing up to its equilibrium point. When the pendulum is close to the upright position, the stabilizing controller designed earlier can be triggered to catch the pendulum and balance it around the equilibrium point.

The algorithm is defined such that the energy E in equation (10) is zero in the upright position. The energy of the pendulum can be written as:

$$E = (m_1 + m_2) g l_c \left[\frac{1}{2} \left(\frac{\dot{\theta}}{w_o} \right)^2 + \cos(\theta) - 1 \right] \quad \text{where, } w_o = \sqrt{\frac{m_2 g l_c}{4I}} \quad (10)$$

The control law implemented to achieve the desire energy is:

$$F = (m_1 + m_2) \left(\text{satv} \left(C(E - E_o) \right) \text{sign}(\dot{\theta} \cos(\theta)) \right) \quad (11)$$

In this equation adopted from Olfati-Saber [2], the parameter C is a design value to improve the controller and E_o is the desired energy level. Combining equation (8) with (11), it is possible to express the required voltage given a torque. From equation (11), the sign of $\dot{\theta} \cos(\theta)$ of the pendulum determines the acceleration and direction of the cart, then the control of the balancing of the pendulum.

For the rotating pendulum, we have adopted the same strategy [10]. Equation (7) shows the nonlinear dynamics model, in order to stabilize the pendulum in the upright position, the linearization of these equations in the equilibrium point, defined as $q_2 = q_1 = 0$ and $\dot{q}_2 = 0$, yields:

$$\begin{aligned} X_1 &= q_2, X_2 = q_1, X_3 = \dot{q}_2, X_4 = \dot{q}_1 \\ \dot{X} &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \left(\frac{I + mr^2}{Il_p} \right) g & 0 & \left(\frac{I + mr^2}{ml_p^2 I} \right) b & \frac{r}{l_p IR_a} \\ -\frac{rmg}{I} & 0 & -\frac{rbs}{Il_p} & -\frac{1}{IR_a} \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ \frac{rK_a}{Il_p R_a} \\ \frac{K_a}{IR_a} \end{bmatrix} V \\ Y &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} X \end{aligned} \quad (12)$$

Note that the dc motor dynamics from equation (8) has been included into the linear state model in (12). The same approach for the swing-up controller applied to cart-pole system has also been considered in the rotating pendulum, there is just one difference related with the motor model in equation (8) and some physical parameters from the system.

III. SOFTWARE DESIGN AND IMPLEMENTATION.

As we mentioned in section two, we have developed a platform named VISUNS in order to simulate all the theory exposed previously. In relation with the underactuated systems, we have developed five predefined mechanical systems embedded into the platform: the cart-pole system, the acrobat and pendubot systems, the rotating pendulum and the inertia wheel pendulum, which are some of the most typical underactuated nonlinear systems examples used to testing non linear control techniques [2]. VISUNS is not tied to these five virtual systems, the paradigm design of the tool allows to programmers add and interconnect other systems easily. For the design of the platform we have identified 3 modules composed by *Graphical Engine*, the *Main Class* and the *Simulation Engine*.

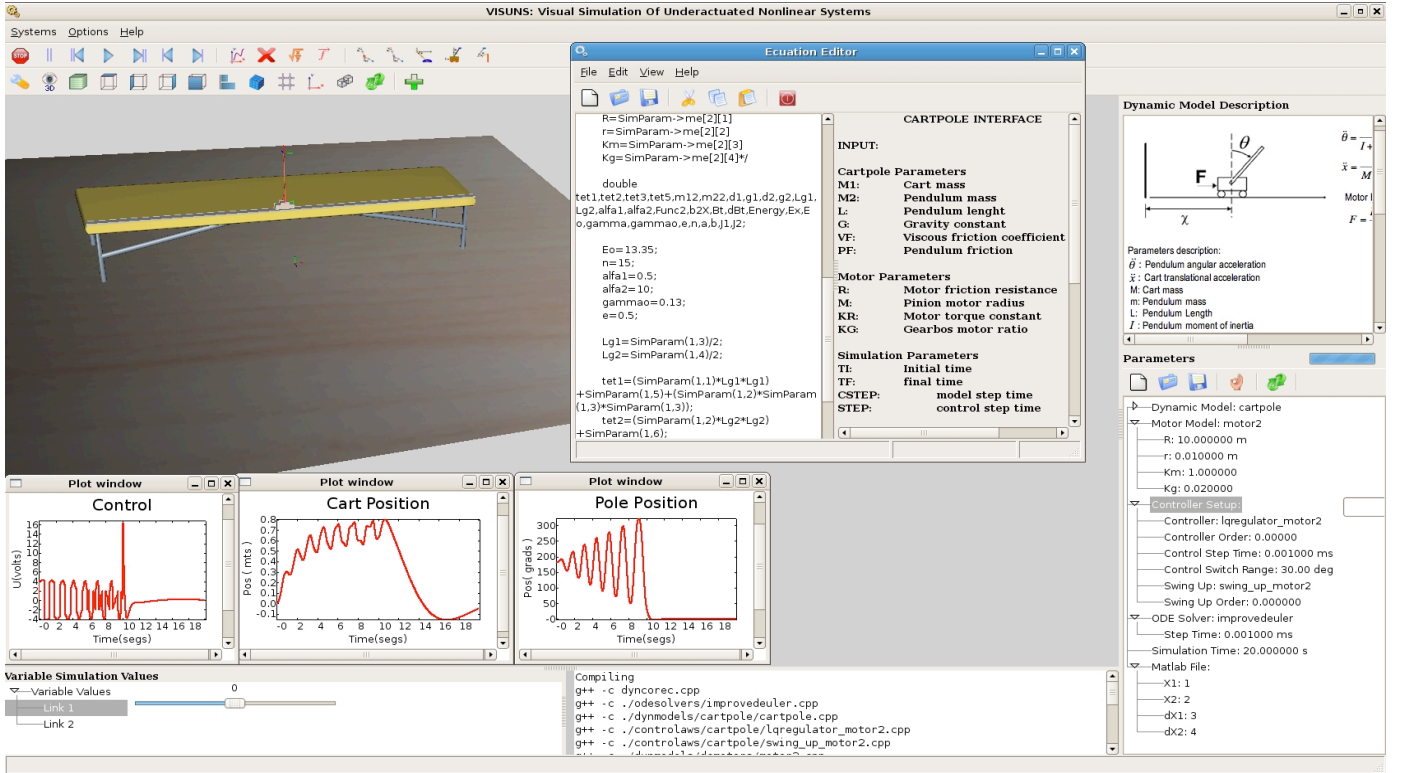


Fig. 2. VISUNS graphical user interface.

The Graphical user interface-GUI, shown in Fig. 2 has been implemented using wxPython, a library with a collection of widgets that allow an easier menus implementation and a comfortable look and feel to the user. The graphical engine was defined using VTK 3-D engine [11], which contains many different classes and primitives for solid rendering with a great collection of graphics and visualization techniques such as illuminations, shadows, textures and stereoscopic vision, which are one of the most amazing visualization feature included into the tool, providing realism to the simulations.

Inside the Main Class, the tool defines the simulation structure given by the user using the graphical interface; the user may load a system, then, modify its parameters such as: mass, inertias, lengths, integration time and step time, the numerical integrator, the predefined controller and the dc-motor model to use or the control law equation given by the user using the interpreter as shown in Fig. 2. This interpreter has a Matlab-like syntax, thus, the user just opens the interpreter template from the menu and writes the control law equations in state variable form and the interpreter will translate and complete the rest. The information about the nonlinear dynamic model is also shown, so the users may use it previously for the design of their own controllers.

IV. SIMULATION AND CONTROLLER RESULTS.

One of the most important aspects of any modeling and simulation software is how well its results correlate to the actual physical behavior of the corresponding simulated

system. In this case, we do not have a real system to validate results, nevertheless, the numerical solutions will be compared with Matlab-Simulink, perhaps, the most used software for modeling and simulating of control systems.

A. The cart-pole system.

The cart-pole system using the nonlinear dynamic model in equation (4) has been tested with: $m_1 = 0.5kg$, $m_2 = 0.2kg$, $l_c = 0.3m$, $b = c = 0$, $K\tau = 1Nm/A$, $Kg = 0.02m$, $R = 10\Omega$, $r = 0.01m$. The initial values are: $X_1 = 0$, $X_2 = 0$, $X_3 = \pi$, $X_4 = 0$, which means that the pendulum will start from its downward position using the swing-up controller from (11) with $C=2$ and with a low desire energy level $E_o = 0.1J$. Once the pendulum swings to its upright position defined as $30^\circ \geq \theta \geq -30^\circ$, a Linear Quadratic Regulator (LQR) will stabilize the pendulum to its equilibrium point defined in $X_3 = 0$. For this, the linear model in (9) was used, obtaining the gain matrix K for a linear state feedback control law $u = -KX$. For the LQR design [12], the weighting parameters Q and R chosen for the optimal state feedback controller and the gain matrix K are:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ with } R = 1 \text{ and } K = \begin{bmatrix} -1 & -77.04 & -4.15 & -12.26 \end{bmatrix}$$

Running the simulations with the initial values established with a step time of 0.001s and a total simulation time of 20s using a fixed step numerical integrator based on *RungeKutta*:

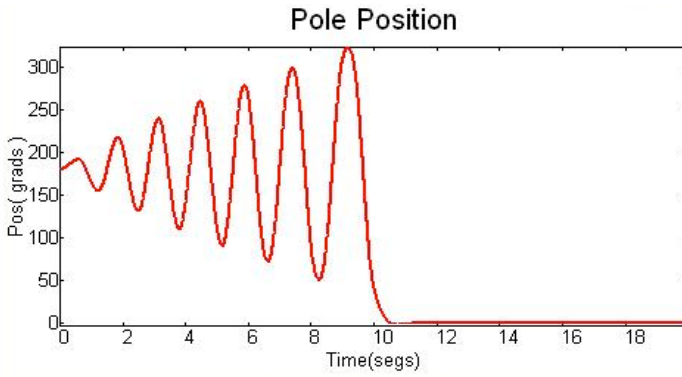


Fig. 3. Swing-up+LQR in VISUNS.

Running the simulation with the same characteristics using Matlab-Simulink, with a fixed step of 0.001s (ODE45 solver):

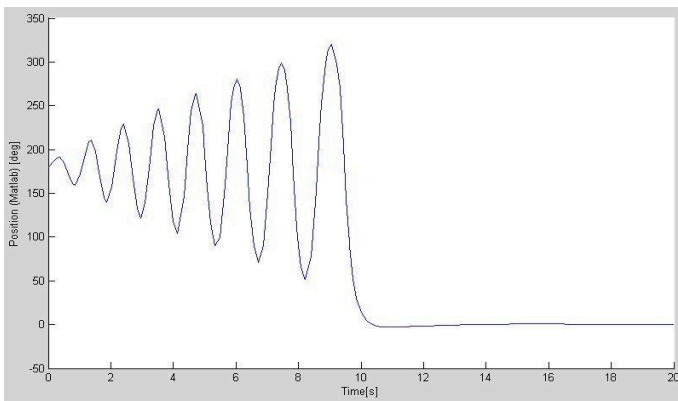


Fig. 4. Swing-up+LQR in Matlab.

The correlation between results in Fig. 3 and Fig. 4 are shown in the following error curve:

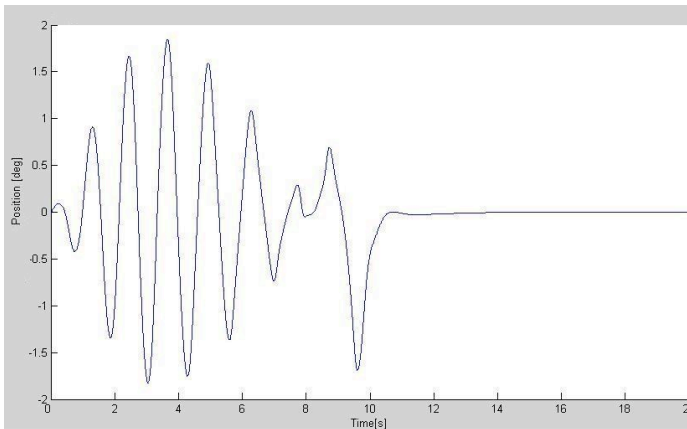


Fig. 5. Deviation curve (Error between VISUNS and Matlab)

Once our dynamic model, controllers and numerical integrator embedded into VISUNS have been validated, other interesting results are analyzed:

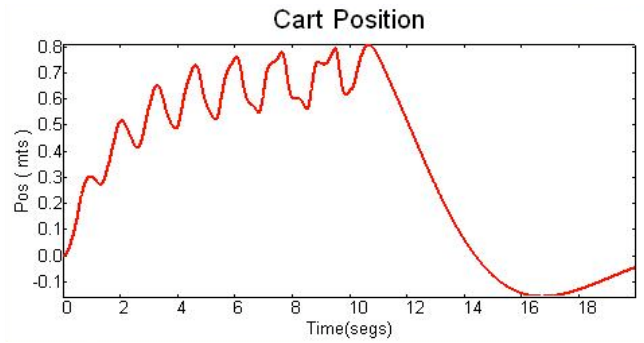


Fig. 6. Cart displacement.

In Fig. 3 the pendulum swings about 10 seconds before LQR stabilizes it. In some cases, this could also be achieved but with a larger cart displacement. Fig. 6 shows a maximum displacement just of 80cm with a control effort between 4 volts and 16.5 volts, as depicted in Fig. 7:

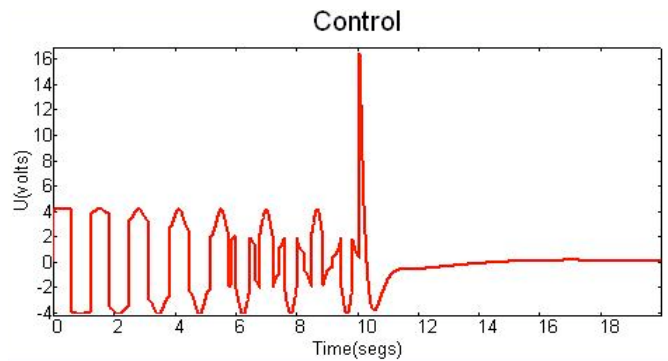


Fig. 7. Cart-pole control effort.

Swing-up control may be saturated using the function *satv* in equation (11), in order to achieve less control effort. We rather do not saturate this function, instead, by tuning the energy parameters E_o in (11). This modification will improve the total response of the system, achieving less control effort in the stabilization area and a faster swing up acceleration to the upright position. In the last section we will explain this approach deeper. With this new analysis, the energy constant is defined as $E_o = 3.5J$ when the pendulum is downwards, but when it is close to the upright position, $E_o = -0.5J$. These values could be established experimentally observing the swing energy as shown in Fig. 8 (note that the energy is zero in the upright position).

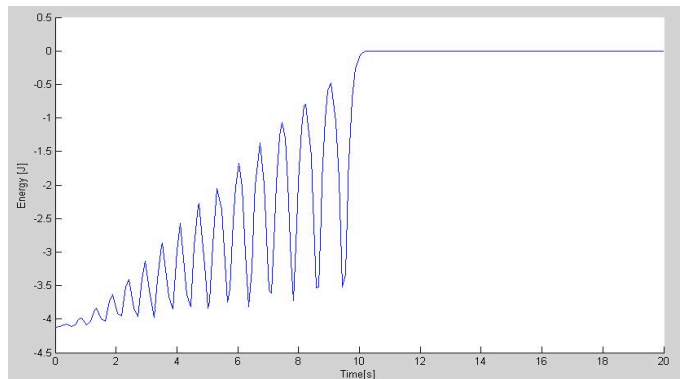


Fig. 8. Swing energy.

The new response of the system corresponds to:

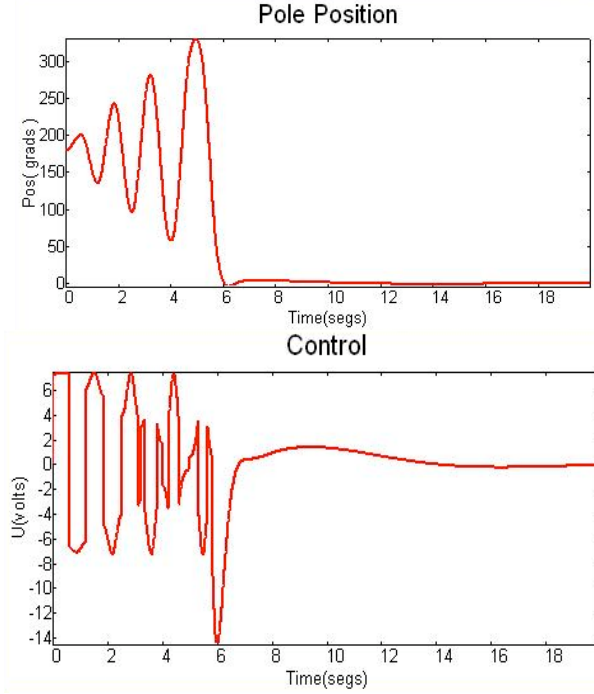


Fig. 9. Improved controller response.

In Fig. 9 the number of pendulum's swings with less voltage to apply was reduced in the stabilization area. The pendulum swings about 4 seconds less than the simulation in Fig. 3 with a maximum voltage of 14v instead of 16.5v. Of course, this is not the best result to obtain, and there are more proofs to do in order to improve the response of the system achieving better performance. The parameters chosen in equation (9) are: $m = 0.2kg$, $l_p = 0.35m$, $I = 0.06kgm^2$, $b = 0Nm/rad/s$, $Ra = 10\Omega$

$r = 0.3m$, $Ka = 1Nm/A$, including the motor equation in (8). For the swing-up controller $C=1$ and $E_o = 10$. The LQR parameters have been the same used for the cart-pole, obviously the matrix Q depends on the variable state. The gain matrix is:

$K = [-1 \quad -70.7821 \quad -2.9414 \quad -12.2060]$. Related with the simulation variables, step and total time have been the same. *Rungekutta* based numerical integrator embedded into VISUNS has been selected for this simulation.

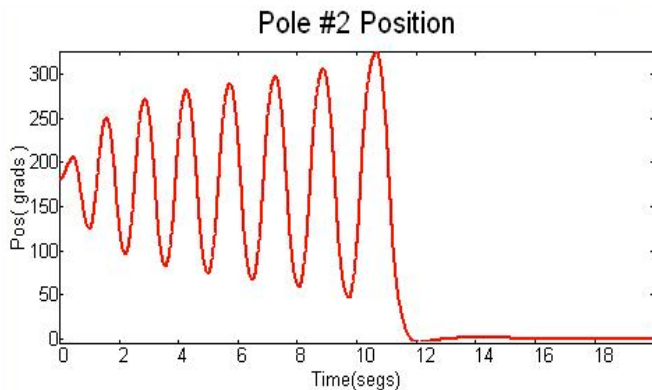


Fig. 10. Rotating pendulum controllers and control effort.

Fig. 10 shows the swing-up and LQR controllers for the rotating pendulum with its respective control effort of maximum 10.5 volts. For the control of the rotating pendulum, it would not be a challenge to stabilize the non-actuated pendulum in the upright position if the actuated rotating joint will have to rotate 360 degrees to swing it. The challenge consists in moving the actuated joint into a specific range in order to swing the pendulum to the upright position as depicted in Fig. 11:

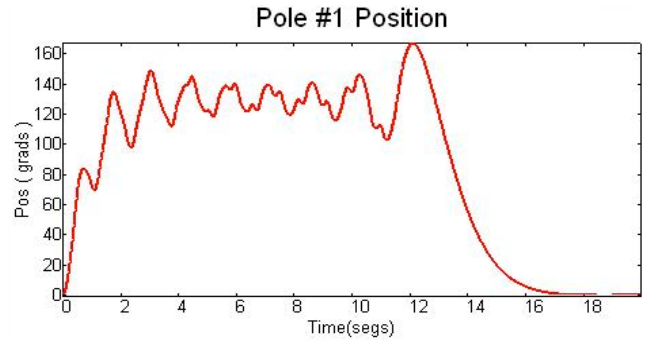


Fig. 11. Actuated link rotation.

V. CONCLUSIONS

The cart-pole and the rotating pendulum systems have been simulated in order to demonstrate the capabilities of the platform developed. Both controllers were capable of successfully swinging a pendulum from an initially downward position to the upright position and balancing the pendulum around that point. As the data indicates, the swing-up energy controller was tuned in order to obtain better performance in relation with response time and control effort in the stabilization area. Looking equation (11), tuning the parameter E_o is possible to know how much energy should be added to the pendulum in order to swing it to its upright position. The tuning of this parameter is heuristic and depends on the swing energy shown in Fig. 8. If E_o is close to be the same to the energy value E from equation (10), the acceleration of the system will be close to zero. Plotting the value of the energy in Fig. 8, when the pendulum is downward to the upright position makes possible to establish how to improve the performance of the controller tuning E_o in order to accelerate the system and to decelerate it in the right time and position. For this fact, E_o ought to be similar to E when

the pendulum is close to the equilibrium point and then guarantee a robust controller in terms of time response and control effort.

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